

TD 3

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February 11, 2019

Problem 1. (2.6.A)

Show that $(a_n) = \left(\frac{n \cos^n(n)}{\sqrt{n^2+2n}}\right)_{n=1}^{\infty}$ has a convergent subsequence.

Problem 2. (2.6.B)

Does the sequence $(b_n) = \left(n + \cos(n\pi)\sqrt{n^2+1}\right)_{n=1}^{\infty}$ have a convergent subsequence?

Problem 3. (2.6.D)

Show that every sequence has a monotone subsequence.

Problem 4. (2.6.H)

Let $(x_n)_{n=1}^{\infty}$ be a sequence of real numbers. Suppose that there is a real number L with the property that every subsequence $(x_{n_k})_{k=1}^{\infty}$ has a subsubsequence $(x_{n_{k(l)}})_{l=1}^{\infty}$ with

$$\lim_{l \rightarrow \infty} x_{n_{k(l)}} = L.$$

Show that the whole sequence converges to L .

Hint. If it were false, then you could find a subsequence bounded away from L .

Problem 5. (2.6.J)

Suppose that $(x_n)_{n=1}^{\infty}$ is a sequence of real numbers. If $L = \liminf x_n$, show that there is a subsequence $(x_{n_k})_{k=1}^{\infty}$ so that $\lim_{k \rightarrow \infty} x_{n_k} = L$

Problem 6. (2.7.A)

Give an example of a sequence (a_n) such that $\lim_{n \rightarrow \infty} |a_n - a_{n+1}| = 0$, but the sequence does not converge.

Problem 7. (2.7.G)

Evaluate the continued fraction $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}}$.