TD 5

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Problem 1. *Let* $f\colon [a,b] \to \mathbb{R}$ *and* $g\colon [b,c] \to \mathbb{R}$ *be continuous functions that agree on the overlap (i.e. such that* $f(b) = g(b)$*. Show that* $h\colon [a,c] \to \mathbb{R}$*, defined by*

$$h(x) = \begin{cases} f(x) & x \in [a,b] \\ g(x) & x \in [b,c] \end{cases}$$

is continuous.

Problem 2. *Assume that the temperature* $T(x)$ *at a point* x *on a sphere of radius 1 is continuous in space, i.e. a continuous function* $T\colon S^2 \to \mathbb{R}$*. Show that there is a point* $y \in S^2$ *on the surface such that* $T(y) = T(-y)$*. Hint: consider* $f(x) = T(x) - T(-x)$ *and compare* $f(x)$ *with* $f(-x)$*.*

Problem 3. *Let* $f\colon \overline{B}(0;1) \to \mathbb{R}$ *be a continuous function, where* $\overline{B}(0;1) \subset \mathbb{R}^2$ *is the closed ball of radius 1, centred at* $(0,0)$*. Show that* f *cannot be injective.*